QSCU 221 Midterm 1 Review Session - Practise Questions

*** This is way longer than your actual midterm will be

1. True or false:
    a. If a system of equations has 2 solutions, then it has infinitely many solutions.
    b. ~~*If A is $m \times n$ and the equation $A\vec{x} = \vec{b}$ is consistent for every $\vec{b}$ in $\mathbb{R}^n$, then rank(A)$=m$~~
    c. If $T: \mathbb{R}^a \to \mathbb{R}^b$ then A is an a x b matrix.
    d. If A is a 4x3 matrix, then the transformation $\vec{x} \mapsto A\vec{x}$ maps $\mathbb{R}^3$ onto $\mathbb{R}^4$.
    e. If A is a 3x2 matrix then the columns of A are linearly independent.
    f. Let $T: \mathbb{R}^n \to \mathbb{R}^k$ be a linear transformation. Then then range of T is $\mathbb{R}^k$.
    g. The matrix $\begin{bmatrix} 1 & 3 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in reduced row echelon form.
    h. If the rightmost column of a coefficient matrix is a pivot column, then the corresponding linear system is consistent.
    i. * Let $\overrightarrow{v_1}, \vec{v}_2 \in \mathbb{R}^k$. Then $\vec{0}$ is in span$\{\overrightarrow{v_1}, \vec{v}_2\}$.
    j. The columns of a 5 x 4 matrix can both span $\mathbb{R}^5$ $and\ be\ linearly\ independent$.

2. Consider a linear system with 231 variables and 417 equations. Let A be the coefficient matrix for the system. Determine if the following statements are **always true, sometimes true,** or **never true.** Justify your answer.
    a. $A\vec{x} = \vec{b}$ does not have a solution for all $\vec{b} \in \mathbb{R}^{417}$.
    b. $\vec{0}$ can be written as a non-trivial linear combination of the columns of A.

3. Write down the augmented matrix of the following echelon system and use the backward phase of the Gauss-Jordan algorithm to find its reduced echelon form. Then write out the solution of the system in parametric vector form.

$$x_1 + 2x_2 - 4x_3 + 2x_4 + 6x_5 = 7$$

$$-2x_3 + x_4 - 3x_5 = 9$$

$$x_4 + 3x_5 = -5$$

4. Define what it means for the vector $\vec{w}$ to be a linear combination of the vectors $\vec{v}_1, \overrightarrow{v_2}, \ldots \overrightarrow{v_k}$.

5. Suppose that A is a 3x3 matrix with <u>exactly 2</u> pivot columns.
    a. Write down all possible <u>reduced</u> row echelon forms of A. (Use * to represent an entry that can have any value)

b. If A is the coefficient matrix of a linear system, must the system be consistent? Yes or no? Why?

6. Consider $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & -2 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. The reduced row echelon form of $[A|\,b]$ is

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

a. Express the solution to $A\vec{x} = \vec{b}$. in parametric vector form
b. Can the vector $\vec{b}$ be written as a linear combination of the columns of A? If so, then find a set of weights that work. If not, explain why.
c. Suppose that $T(\vec{x}) = A\vec{x}$ is the linear transformation whose standard matrix is the matrix given above. What are the domain and the codomain of T?
d. Find the image of $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ under T.
~~e. Does the vector b above have a pre-image under T? If so, find it. If not, explain why.~~

7. Let $\vec{x}_1$ and $\overrightarrow{x_2}$ be solutions of $A\vec{x} = \vec{b}$. Show that $\vec{x}_1 - \vec{x}_2$ is a solution of $A\vec{x} = \vec{0}$.

8. Let A be an n x m matrix and let $\vec{b} \neq \vec{0}$ be a non-zero vector in $\mathbb{R}^n$, and suppose that vectors u, v, and w in $\mathbb{R}^m$ are any 3 solutions for the equation $A\vec{x} = \vec{b}$. Is the vector $\vec{p} = 2\vec{u} - 4\vec{v} + 3\vec{w}$ a solution of the homogeneous equation $A\vec{x} = \vec{0}$? Yes or no? Use matrix-vector algebra to justify your answer.

9. Let $A = [\vec{a}_1\ \vec{a}_2\ \vec{a}_3\ \vec{a}_4]$ be a 4x4 matrix and suppose that $\vec{x} = \begin{bmatrix} 6 \\ -3 \\ 0 \\ 2 \end{bmatrix}$ is a solution of the homogeneous equation $A\vec{x} = \vec{0}$..
   a. Give a linear dependence relationship on the columns of A.
   b. Do the columns of A span $\mathbb{R}^4$? Why or why not?

10. Let A be a 12x7 matrix.
   a. Does the system $A\vec{x} = \vec{b}$ have a solution for every b in $\mathbb{R}^{12}$? Why or why not?
   b. Are the columns of A linearly independent? Why or why not?

11. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+b \\ b+c \\ c+a \end{bmatrix}$. Find the standard matrix of T.

12. Show that $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} a \\ 2 \end{bmatrix}$ is not linear.

13. Is the set $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ linearly independent or dependent? If the vectors are dependent, then write down a specific linear dependence relation for them.

14. Suppose the system below is consistent for all possible values of s and t. What can you say about the coefficients c and d? Justify your answer.

$$x_1 + 2x_2 = s$$
$$cx_1 + dx_2 = t$$

15. Determine all values of k (if any) such that the vector $\vec{b} = \begin{bmatrix} 2 \\ k \\ 5 \end{bmatrix}$ is in the plane given by $span\left\{\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}\right\}$.

16. Let $T: \mathbb{R}^3 \to \mathbb{R}$ be a linear transformation. Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ be two non-zero vectors that satisfy $T(\vec{u} + 2\vec{v}) = 5, T(-\vec{u} - \vec{v}) = -5$.
   a. Use the above information to calculate $T(\vec{v})$.
   b. Explain using the definition of one-to-one why T is not a one-to-one linear transformation.